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# HEAT-TREATMENT PROCESSES IN THE FORMATION OF COMPONENTS FROM POLYMER COMPOSITE MATERIALS

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A physicomathematical model of the heat treatment of components made from polymer composite materials on a rotating mandrel is considered.

With the expanding use of polymer composite materials (PCM) of constructional specification in various branches of the economy, the problem of retaining the initial properties of the components in the PCM is of great urgency. For example, in fiber-glass constructions obtained by the rolling method, only one third of the strength of the reinforcing fiber may be realized in practice [1]. In connection with this, it is necessary, first of all, to ensure optimal conditions for performing all stages of the technological process of producing PCM components: preparation of the initial components; soaking; formation of the components; solidification of the binder, etc.

The important factors influencing the final properties of the material obtained are the methods employed and the technology for thermal hardening of the polymer binder. The first largely determines the relative homogeneity of development of the temperature fields over the cross section of the component which forms, and the second the temperature—time characteristics of the process, which must ensure correspondence between the energy densities supplied and the physicochemical changes in composition occurring.

In realizing intensive conditions of PCM heat treatment in production, and in constructing and creating units in which effective thermoradiational—convective methods of energy supply are used, it is necessary to ensure "gradientless" (over the cross section of the component) heating in order to obtain high-quality components by methods of both wet and dry winding. In conection with the difficulty of experimental determination of temperature fields in PCM components, solidified on rotating mandrels, the mathematical modeling of such processes takes on practical importance.

The heat-conduction problem for systems of two cylindrical bodies is solved: components in the form of PCM tubes wound on a mandrel, rotating at specified frequency $\omega$. The radiation sources are distributed on an arc s, as a result of which the component is subjected to pulsed heating. It is assumed that, if a point on the body falls under the radiation of the sources on arc s, then the pulse function $\varphi(\tau) = 1$, whereas if the point is in shadow then $\varphi(\tau) = 0$.

The mathematical model of this process comprises a system of equations of the form

$$\frac{\partial T}{\partial \tau} = a_2\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right), \quad R_3 \leqslant r \leqslant R_2, \tag{1}$$

$$\frac{\partial T}{\partial \tau} = a_1\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right), \quad R_2 \leqslant r \leqslant R_1, \tag{2}$$

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with matching conditions at the boundary $R_2$

$$T(R_2+0,\ \tau)=T(R_2-0,\ \tau), \tag{3}$$

$$\lambda_2\frac{\partial T(R_2-0,\ \tau)}{\partial r}=\lambda_1\frac{\partial T(R_2+0,\ \tau)}{\partial r}. \tag{4}$$

The heat flux at the external boundary of the body is determined by the constructional parameters of the technological equipment for heat treatment, and is specified by an equation containing the pulse function

$$\lambda_1\frac{\partial T(R_1,\ \tau)}{\partial r}=Aq_N\varphi(\tau)-\alpha[T(R_1,\ \tau)-T_{av}]. \tag{5}$$

For the sake of generality, the case of a hollow mandrel with closed ends is considered. The heat consumption in heating the air inside the mandrel is negligibly small in comparison with the heat consumption in heating the mandrel itself; therefore, it may be supposed that there are no heat sources at the internal boundary, that is

$$\frac{\partial T(R_3,\ \tau)}{\partial r}=0. \tag{6}$$

Initially, the temperature is constant

$$T(r,\ 0)=T_0,\quad R_3\leqslant r\leqslant R_1. \tag{7}$$

The time of pulse action is calculated as follows. Let $\tau_2 = \omega^{-1}$ denote the cycle (period of revolution) and $\tau_1 = s\omega^{-1}$ the time of irradiation in a single revolution. Then the pulse function in the n-th revolution may be specified according to the rule

$$\varphi(\tau)=\begin{cases}1, & n\tau_2\leqslant\tau\leqslant n\tau_2+\tau_1,\\ 0, & n\tau_2+\tau_1\leqslant\tau\leqslant(n+1)\tau_2.\end{cases} \tag{8}$$

The problem in Eqs. (1)-(8) may be solved using an implicit difference scheme constructed on a grid of points with spacing $h_1 = (R_1 - R_2)/N_1$ in the region $R_2 \le r \le R_1$, spacing $h_2 = (R_3 - R_2)/(N_2 - N_1)$ in the region $R_3 \le r \le R_2$, and spacing $h_\tau > 0$ with respect to the time variable $\tau$. The approximation is undertaken by means of a difference scheme of the form

$$\begin{gathered}\frac{\hat{y}-y}{h_\tau}=a_1(\hat{y}_{\bar{r}r}+r^{-1}\hat{y}_{\overset{\circ}{r}}),\quad R_2<r<R_1,\\ \frac{\hat{y}-y}{h_\tau}=a_2(\hat{y}_{\bar{r}r}+r^{-1}\hat{y}_{\overset{\circ}{r}}),\quad R_3<r<R_2,\\ \lambda_1\hat{y}_r=\lambda_2\hat{y}_{\bar{r}},\quad r=R_2,\quad \lambda_1\hat{y}_r=Aq_N\varphi(\tau)-\alpha(\hat{y}-T_{av}),\ r=R_1,\\ \hat{y}_r=0,\ r=R_3,\ y(r,\ 0)=T_0,\ R_3\leqslant r\leqslant R_1,\end{gathered} \tag{9}$$

where y is the approximate solution of the initial problem. All the notation in Eq. (9) is conventional in difference-scheme theory, and is taken from [2].

The realization, the difference scheme in Eq. (9) is reduced to a form suitable for the use of difference fitting [2]

$$\begin{gathered}A_i\hat{y}_{i-1}-C_i\hat{y}_i+B_i\hat{y}_{i+1}=-F_i,\quad i=\overline{1,\ N_2-1},\\ \hat{y}_0=\varkappa_1\hat{y}_1+\nu_1,\quad \hat{y}_{N_2}=\varkappa_2\hat{y}_{N_2-1}+\nu_2,\end{gathered} \tag{10}$$

where

$$A_i=a_1\left(1-\frac{h_1}{2(R_3+ih_1)}\right);\quad B_i=a_1\left(1+\frac{h_1}{2(R_3+ih_1)}\right);$$

$$C_i=2a_1+\frac{h_1}{h_\tau},\quad i=\overline{1,\ N_1-1};$$

$$A_i = a_2\left(1 - \frac{h_2}{2(R_3 + ih_2)}\right); \quad B_i = a_2\left(1 + \frac{h_2}{2(R_3 + ih_2)}\right);$$

$$C_i = 2a_2 + h_2/h_\tau, \quad i = \overline{N_1+1,\ N_2-1};$$

$$A_{N_1} = \lambda_2 h_1; \quad B_{N_1} = \lambda_1 h_2 + \lambda_2 h_1; \quad C_{N_1} = \lambda_1 h_2;$$

$$F_i = y_i h_1^2 h_\tau^{-1}, \quad i = \overline{1,\ N_1 - 1}; \quad F_{N_1} = 0; \quad F_i = y_i h_2^2 h_\tau^{-1}, \quad i = \overline{N_1+1,\ N_2-1};$$

$$\varkappa_1 = 1; \quad \nu_1 = 0; \quad \varkappa_2 = \lambda_1(\lambda_1 + h_1\alpha)^{-1};$$

$$\nu_2 = \frac{h_1}{\lambda_1 + \alpha h_1}\left(Aq_N\varphi(\tau) + \alpha T_{av}\right).$$

The system in Eq. (10) satisfies the condition of stability of the difference-fitting method, since with any steps $h_1$, $h_2$, $h_\tau$, the difference grids $A_i$, $B_i$, $C_i$ are positive, and

$$C_i \geqslant A_i + B_i, \quad |\varkappa_1| + |\varkappa_2| < 2.$$

The temperature fields are calculated as follows. Initially, the temperature over the whole depth is assumed to be equal to the initial value $y(x, 0) = T_i^0 = T_0$, $R_3 \le r \le R_1$, $i = 0, \ldots, N_2$. The first revolution on rotation begins with an irradiation zone. The time of revolution $\tau_F - \tau_I = \omega^{-1}$ min $= 60\ \omega^{-1}$ sec, where $\tau_F = (n + 1)\omega^{-1}$ and $\tau_I = n\omega^{-1}$ is divided into $N\tau$ equal intervals with step $h_\tau = (\tau_F - \tau_I)/N_\tau$, and on the segment $[\tau_I, \tau_F]$ there are points $\tau_j = \tau_I + jh_\tau$, $j = 0, \ldots, N_\tau$. Let $\tau_n$ denote the time at which the cross section of the body reaches the shadow in each revolution. Then it is simple to show that $\tau_n = \tau_I + s\cdot 60/\omega$. Hence it follows that in Eq. (10), when $\tau_j \in [\tau_I, \tau_n]$, the pulse function $\varphi(\tau) = 1$, and when $\tau_j \in [\tau_n, \tau_F]$, the pulse function $\varphi(\tau) = 0$.

Thus, in any revolution n, at time $\tau_{j+1}$, solving the system of linear algebraic equations in Eq. (10), the temperature distribution over the whole depth of the component is obtained:

$$\hat{y} = y_i^{j+1} = T(r_i, \tau_{j+1}), \quad i = \overline{0,\ N_2}, \quad j = \overline{0,\ N_\tau}.$$

By special introduction of the norms of the grid solution of the form

$$\|W^{j+1}\|^2 = \|\hat{Z}\|^2 + (1 + \tau C_*)^{-1}(\tau_{a_*}\|\hat{Z}_{\bar{r}}\|^2 + \tau^2\|Z_t\|^2 + 2\tau a_1 a_2 \alpha\lambda_2 \hat{Z}_{N_2}^2$$

and use of the energy-inequality method, the convergence of the solution of the difference problem in Eqs. (1)-(7) may be proven; this coincides in order of magnitude with the error of the difference-scheme approximation. The following estimate is valid here

$$\|W^{j+1}\|^2 \leqslant e^{t_j C^*}(1 - \tau C_*)\, t_j \|\psi_*\|^2 = O(h_\tau + h_1 + h_2),$$

where $\psi_*$ is the error of the approximation; $a_*$, $C_*$ are constants.

Numerical experiments are undertaken with variation of the thermophysical and technological parameters in the ranges: $\lambda_1 = 0.4$-$1.0$ W/m·K, $a_1 = 2\cdot 10^{-7}$ m²/sec; $\lambda_2 = 0.4$-$50$ W/m·K, $a_2 = 2\cdot 10^{-7}$-$1.6\cdot 10^{-5}$ m²/sec, $R_1 = 0.2$ m, $R_1 - R_2 = 0.001$-$0.020$ m, $R_2 - R_3 = 0.008$ m, $s = 0.3$-$1$, $\omega = 0.1$-$10$ rpm, $T_0 = 293$ K, $T_{av} = 293$-$423$ K, $\alpha = 10$-$20$ W/m²·K, $q_N = 1000$-$5000$ W/m².

Some calculation results are shown in Figs. 1-3. As is evident from Fig. 1, the temperature of the external surface of the composite varies cyclically in accordance with the rotational period of the mandrel, and takes a sawtooth form. The temperature of the composite surface in contact with the mandrel increases smoothly. Variation in rotational frequency of the mandrel in the range $\omega = 1$-$5$ rpm has practically no influence on the kinetics of heating of a composite of thickness 2 mm. However, with increase in $\omega$, other conditions being equal, the temperature difference over the surface in the heating period and in steady conditions decreases by a factor of 2-2.5.

The thickness of the composite material has the greatest influence on the heating kinetics. Thus, with a heat-flux density $q_N = 1000$ W/m², the steady temperature $T_{st}$ is reached after 7200, 124,000, and 24,000 sec, respectively, for thicknesses $\delta = 2$, 5, and 20 mm. At the same time, the limiting temperature of the material, other conditions being equal, does not greatly depend on the thickness (Fig. 2).

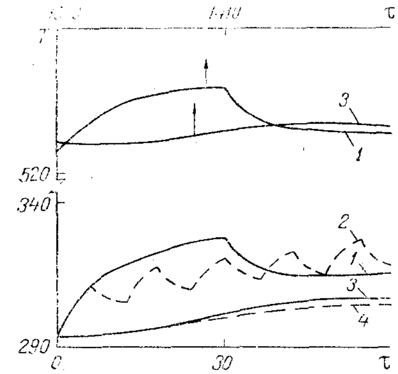


Fig. 1

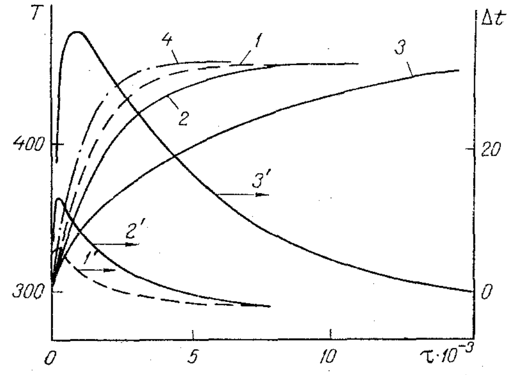


Fig. 2

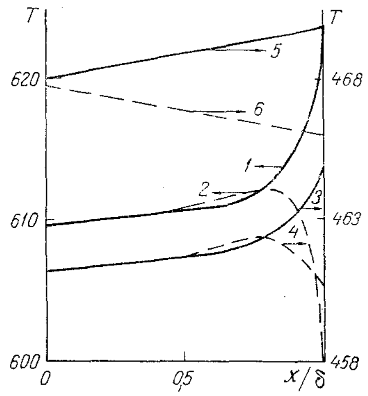


Fig. 3

Fig. 1. Kinetic curves of the heating of the irradiated composite surface (1, 2) and the surface in contact with the mandrel (3, 4): 1, 3) ω = 1; 2, 4) 5 rpm. T, K; τ, sec.

Fig. 2. Influence of the composite-material thickness on the heating kinetics of the composite material; 1, 1', 4) δ = 2; 2, 2') 5; 3, 3') 20 mm; 4) heating on a mandrel of composite material. $\tau \cdot 10^{-3}$, sec.

Fig. 3. Influence of the thickness and heat-flux density on the temperature field in the composite: 1-4) δ = 20 mm; 5, 6) 2 mm; 1, 2) $q_N$ = 5000 W/m²; 3-6) 1000 W/m²; 1, 3, 5) end of irradiation; 2, 4, 6) end of revolution.

On a mandrel of material with a low thermal conductivity (for example, a composite material), the component is more rapidly heated than on a metallic mandrel, other conditions being equal; this is because the mandrel of composite material accumulates heat more slowly. However, the limiting heating temperature does not depend on the mandrel temperature (Fig. 2).

With fivefold increase in the heat-flux density (from 1000 to 5000 W/m²), other conditions being equal, the temperature in steady conditions is increased from 460 to 605 K. The temperature difference over the thickness of the composite $\Delta T = T(R_1, \tau) - T(R_2, \tau)$ changes here from −1.5 to −10.6 K at the end of revolution; at the end of irradiation it changes from 3.5 to 14.5 K, i.e., the temperature oscillations over the thickness are 5 and 25 K, respectively, over the course of a revolution, when $q_N$ = 1000 and 5000 W/m². The temperature of the external surface also varies within the same limits in the course of a revolution, at the given flux densities.

Over the whole heating process, the temperature difference over the shell thickness decreases. Thus, whereas ΔT is greater than 40 K at the onset of heating when $q_N$ = 1000 W/m² and δ = 20 mm, in steady conditions it is no more than 4 K. The overheating of the irradiated surface after the revolution in this case is no more than 5 K. With increase in irradiation of the surface ($q_N$ = 5000 W/m²), the temperature difference over the shell thickness in the heating period reaches 86 K on leaving the irradiation zone and 62 K at the end of the revolution. In steady conditions, $\Delta T$ is 13 K at the end of irradiation while at the end of the revolution $\Delta T = -12$ K, i.e., changes sign.

Characteristic curves of the variation in temperature difference over the thickness of the composite shell in steady temperature conditions ($T_1 > T_{av}$) after one revolution are shown in Fig. 3. When $\delta = 20$ mm, the temperature at the end of the revolution increases over a depth approximately equal to 0.25 of the thickness away from the external shell surface, and then decreases to the shell–mandrel contact boundary, i.e., the temperature profile over the composite thickness takes the form of a curve with a maximum close to the outer irradiated surface.

As is evident from Fig. 3 (curves 5 and 6), in steady conditions, in a shell of thickness $\delta = 2$ mm, the temperature variation occurs in the course of a revolution over the whole thickness. With increase in $\delta$ and $\omega$, the limit of the temperature variations is shifted to the outer surface.

Analysis of the numerical experiments shows that the model proposed permits the choice of optimal technological and constructional parameters ensuring the heat-treatment conditions of composite materials with permissible temperature differences, especially if it is difficult to perform experimental temperature measurements, for example, as in the case of mandrel revolution here considered. Thus, using well-founded methods of energy supply and heat-treatment conditions of the wound components, directional influence may be exerted on the qualitative characteristics of composite materials, within known limits.

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# POSITIVE COLUMN OF GLOW DISCHARGE IN LONGITUDINAL GAS FLOW

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UDC 537.525

A system of nonlinear equations describing a positive discharge column in a cylindrical channel with a gas flow is solved.

It is known that a gas flow has a positive influence on the characteristics of a glow discharge (GD): It allows the energy contribution to the discharge to be increased and ensures high stability [1-3]. In connection with this, the development of methods of calculating GD characteristics in a gas flow is an urgent problem. In the present work, an analytical solution of the problem of the positive column (PC) of a discharge is obtained, taking into account that the processes are nonsteady.

Suppose, as in the theory of a discharge with no flow in diffuse conditions [4], that the electron temperature is constant over the channel cross section, ionization occurs from the ground state of the atom by single-electron impact, and volume recombination, diffusion along PC, and convective transfer in the radial direction may be neglected in comparison with ambipolar diffusion, volume ionization by electron impact, and convective transfer along the channel axis. The degree of ionization in the PC region is sufficiently small ($n_e/N \leq 10^{-5}$); therefore, the frequency of collisions between charged particles is considerably less than the collision frequency of charged particles and neutral components of the gas. In connection with this, the thermal conductivity, specific heat, and gas density are determined basically by the properties of neutral particles [5]. These simplifying assumptions for the case of weakly ionized GD plasma are physically justified, have been discussed

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